Influence of Internal and Constructional Damping on Vibrations of Telescopic Boom of Truck Crane

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Abstract

This study formulates and solves the problem of transverse damped vibrations in a crane boom of a truck-mounted crane. Dissipation of vibration energy in the model adopted in the study occurs as a result of internal damping of the viscoelastic material (rheological Kelvin-Voigt model) of the beams that model the system and motion resistance in slides between the moving components of the crane boom. Damped frequencies of vibrations and degree of amplitude decay were calculated. The study also presents eigenvalues of system vibration with respect to changes in damping coefficients and system geometry for selected rigidity of springs.

Keywords: internal damping, crane boom, slides, constructional damping

1. Introduction

The investigations concerning vibration in a system for changing the boom radius in truck-mounted cranes has been carried out in numerous studies [Chin, Nayfeh and Abdel-Rahman 2001, Sochacki and Tomski 1999, Kilicaslan, Balkan and Ider 1999, Spruogis, Jakštas, Turla, Ilijin and Šešok 2011, Sochacki 2007]. The effect of small internal and external damping on stability of non-conservative beam systems was presented in [Kirillov and Seyranin 2005]. Gürgöze, Doğruoğlu, and Zeren [2007] demonstrated the effect of internal damping on vibrations of a support beam with a mass attached to the free end of the beam. This study presents the effect of internal and constructional damping on the crane boom transverse vibrations. In the replacement model studied, the actual three-component truck-mounted crane (DST0285) was modelled in the form of four beams (Bernoulli-Euler model) connected with elastic-damping systems (Kelvin-Voigt model) that represent dynamic properties of slides. The dissipation of vibration energy in the model adopted occurs as a result of simultaneous internal damping of the viscoelastic material of the beam used in the model and from motion resistance in slides between the moving components of the crane boom. The results obtained were presented by means of three-dimensional presentations.
2. Physical and mathematical models of the system

The physical model of a telescopic crane boom is presented in Fig. 1. The model takes into consideration the elasticity of slides by modelling the slides with springs with elasticity constants \( k_1 = k_2 \) [N/m]. Damping in the slides of the crane boom was modelled using translational dampers denoted as \( R_1 = R_2 \) [Ns/m]. Viscoelasticity of the material was characterized by Young's modulus \( E_{mn} \) and viscosity coefficients \( E_{*mn} \) of the beams used in the model. The cylinder for changes in the crane was modelled with the spring with elasticity \( k_S \) [N/m].

![Fig. 1. Physical model of telescopic crane boom](image)

Equations of motion for individual beams in the model were denoted as:

\[
E_{mn} J_{mn} \frac{\partial^4 W_{mn}(x,t)}{\partial x^4} + E_{*mn} J_{mn} \frac{\partial^3 W_{mn}(x,t)}{\partial x^3 \partial t} + \rho_{mn} A_{mn} \frac{\partial^2 W_{mn}(x,t)}{\partial t^2} = 0 \quad (1)
\]

where: \( W_{mn}(x,t) \) – transverse displacements of beams
\( E_{mn} \) – Young's modulus for individual beams,
\( E_{*mn} \) – viscosity coefficients in beams,
\( A_{mn} \) – cross-sectional areas of the beams,
\( J_{mn} \) – moment of inertia in beam cross-sections,
\( \rho_{mn} \) – beam material densities,
\( l_{mn} \) – lengths of individual beams,
\( x \) – spatial coordinate,
\( t \) – time,
\( m, n = 1, 2 \)
Solutions for the equations (1) are given by:

$$w_{mn}^V (x) - \gamma_{mn} w_{nn}^II (x) = 0$$  \hspace{1cm} (2)

where: \(\omega^*\) – integrated eigenvalue of the system, \(\omega^* = Re(\omega^*) + iIm(\omega^*), i = -1^{1/2}\)

Substitution of (2) with (1) yields:

$$w_{mn}^IV (x) - \gamma_{mn} w_{nn}^II (x) = 0$$  \hspace{1cm} (3)

where:

$$\gamma_{mn} = \frac{\rho_{mn} A_{nn} \omega^{*2}}{(E_{nn} + iE_{nn} \omega^*) J_{nn}}, \quad \lambda_{mn} = \sqrt{\gamma_{mn}}$$  \hspace{1cm} (4)

The geometrical and natural boundary conditions are as follows:

$$w_1(0) = 0, \quad (E_{11} + iE_{11} \omega^*) J_{11} w_{11}^II (0) = 0, \quad w_1(l_1) = w_{12}(0),$$

$$w_1'(l_1) = w_{12}(0), \quad (E_{21} + iE_{21} \omega^*) J_{21} w_{21}^II (0) = 0,$$

$$w_{21}(l_1) = w_{22}(0), \quad w_2(l_1) = w_{22}(0),$$

$$(E_{11} + iE_{11} \omega^*) J_{11} w_{11}^II(l_1) = (E_{12} + iE_{12} \omega^*) J_{12} w_{12}^II (0),$$

$$(E_{11} + iE_{11} \omega^*) J_{11} w_{11}^II(l_1) - (E_{12} + iE_{12} \omega^*) J_{12} w_{12}^II (0) +$$

$$-(iR_1 \omega^* + k_1) [w_1(l_1) - w_2(l_1)] - k_s w_1(l_1) +$$

$$(E_{21} + iE_{21} \omega^*) J_{21} w_{21}^III (0) = 0, \quad (E_{12} + iE_{12} \omega^*) J_{12} w_{12}^III (l_1) = 0,$$

$$(E_{12} + iE_{12} \omega^*) J_{12} w_{12}^III (l_1) - (iR_2 \omega^* + k_2) [w_1(l_1) - w_2(l_1)] +$$

$$-(E_{21} + iE_{21} \omega^*) J_{21} w_{21}^III (l_1) = 0,$$

$$(E_{21} + iE_{21} \omega^*) J_{21} w_{21}^III (0) - (iR_1 \omega^* + k_1) [w_1(l_1) - w_2(l_1)] +$$

$$(E_{12} + iE_{12} \omega^*) J_{12} w_{12}^III (0) = 0, \quad (E_{22} + iE_{22} \omega^*) J_{22} w_{22}^III (l_2) = 0,$$

$$(E_{22} + iE_{22} \omega^*) J_{22} w_{22}^III (l_2) = 0,$$

$$(E_{21} + iE_{21} \omega^*) J_{21} w_{21}^III (l_2) = (E_{22} + iE_{22} \omega^*) J_{22} w_{22}^III (0),$$

$$(E_{21} + iE_{21} \omega^*) J_{21} w_{21}^III (l_2) - (E_{22} + iE_{22} \omega^*) J_{22} w_{22}^III (0) +$$

$$-(iR_2 \omega^* + k_2) [w_2(l_2) - w_1(l_2)] - (E_{12} + iE_{12} \omega^*) J_{12} w_{12}^III (l_2) = 0.$$  \hspace{1cm} (5)
The solution for the equations (3) is given by:

\[ w_{mn}(x) = C_{1mn}e^{\lambda_{mn}x} + C_{2mn}e^{-\lambda_{mn}x} + C_{3mn}e^{i\lambda_{mn}x} + C_{4mn}e^{-i\lambda_{mn}x} \]  \hspace{1cm} (6)

Substitution of (6) to (5) yields a homogeneous system of equations with respect to unknown constants \( C_{jm\nu} \), which, in the matrix form, can be written as:

\[ [A(\omega^*)] C = 0 \]  \hspace{1cm} (7)

where:

\[ A(\omega^*) = \left[ e_{pq} \right], \quad (p,q = 1,2,16), \quad C = \left[ C_{jm\nu} \right]^T, \quad j = 1,2 - 4 \]  \hspace{1cm} (8)

The system has a non-trivial solution if the determinant of the coefficient matrix (with constant \( C_{jm\nu} \)) equals zero.

\[ \det A(\omega^*) = 0 \]  \hspace{1cm} (9)

Finding integrated eigenvalues of the matrix \( A(\omega^*) \) leads to determination of damped frequencies and the degree of amplitude decay in the system studied. In this study, the real part \( Re(\omega^*) \) of the solution corresponds to the damped vibration, whereas the imaginary part \( Im(\omega^*) \) characterizes the degree of vibration amplitude decay. Presentation of the results was based on positive values of the real and imaginary parts of solutions.

3. Results of calculations

Calculations were carried out for a telescopic truck-mounted crane boom (DST0285). The parameters of the model (Fig. 1) which were used for computations are presented in Table 1. Geometrical and material data of the crane are presented in Table 2.

<table>
<thead>
<tr>
<th>Wielkość:</th>
<th>Value:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_{11}J_{11} = E_{12}J_{12} [\text{Nm}'] )</td>
<td>( 1,045 \times 10^8 )</td>
</tr>
<tr>
<td>( E_{21}J_{21} = E_{22}J_{22} [\text{Nm}'] )</td>
<td>( 1,449 \times 10^7 )</td>
</tr>
<tr>
<td>( \rho_{11}A_{11} = \rho_{12}A_{12} [\text{kg/m}] )</td>
<td>66,511</td>
</tr>
<tr>
<td>( \rho_{21}A_{21} = \rho_{22}A_{22} [\text{kg/m}] )</td>
<td>70,369</td>
</tr>
<tr>
<td>( l_{11} + l_{12} [\text{m}] )</td>
<td>7.95</td>
</tr>
<tr>
<td>( l_{21} + l_{22} [\text{m}] )</td>
<td>variable</td>
</tr>
<tr>
<td>( k_s [\text{N/m}] )</td>
<td>( 5,549 \times 10^5 )</td>
</tr>
</tbody>
</table>
Table 2. Geometrical and material data adopted for computations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of the fixed component of the crane boom [m]</td>
<td>7.95</td>
</tr>
<tr>
<td>Length of the retractable component of the second crane boom [m]</td>
<td>8.3</td>
</tr>
<tr>
<td>Length of the retractable component of the third crane boom [m]</td>
<td>8.2</td>
</tr>
<tr>
<td>Total crane boom length $L_c$ [m]</td>
<td>variable</td>
</tr>
<tr>
<td>External height of the basic component of the crane boom [m]</td>
<td>0.596</td>
</tr>
<tr>
<td>External height of the second component of the crane boom [m]</td>
<td>0.517</td>
</tr>
<tr>
<td>External height of the third component of the crane boom [m]</td>
<td>0.448</td>
</tr>
<tr>
<td>Internal height of the basic component of the crane boom [m]</td>
<td>0.585</td>
</tr>
<tr>
<td>Internal height of the second component of the crane boom [m]</td>
<td>0.509</td>
</tr>
<tr>
<td>Internal height of the third component of the crane boom [m]</td>
<td>0.441</td>
</tr>
<tr>
<td>External width of the basic component of the crane boom [m]</td>
<td>0.397</td>
</tr>
<tr>
<td>External width of the second component of the crane boom [m]</td>
<td>0.355</td>
</tr>
<tr>
<td>External width of the third component of the crane boom [m]</td>
<td>0.311</td>
</tr>
<tr>
<td>Internal width of the basic component of the crane boom [m]</td>
<td>0.39</td>
</tr>
<tr>
<td>Internal width of the second component of the crane boom [m]</td>
<td>0.348</td>
</tr>
<tr>
<td>Internal width of the third component of the crane boom [m]</td>
<td>0.304</td>
</tr>
<tr>
<td>Cylinder outer diameter [m]</td>
<td>0.277</td>
</tr>
<tr>
<td>Cylinder inner diameter [m]</td>
<td>0.25</td>
</tr>
<tr>
<td>Piston outer diameter [m]</td>
<td>0.16</td>
</tr>
<tr>
<td>Piston inner diameter [m]</td>
<td>0.128</td>
</tr>
<tr>
<td>Material density in boom crane and cylinder $\rho_{mn}$ [kg/m$^3$]</td>
<td>7860</td>
</tr>
<tr>
<td>Density of the liquid in the cylinder [kg/m$^3$]</td>
<td>890</td>
</tr>
<tr>
<td>Young modulus for the material of the boom crane and cylinder $E_{mn}$ [Pa]</td>
<td>$2.1 \times 10^{11}$</td>
</tr>
<tr>
<td>Liquid shear modulus [Pa]</td>
<td>$1.25 \times 10^9$</td>
</tr>
</tbody>
</table>

A dimensionless construtional damping coefficient $\mu$, internal damping coefficient of viscoelastic material of beams $\eta$ and elasticity constant $K$ were adopted for presentation of the results.

$$
\mu = \mu_1 = \mu_2 = \frac{R_1}{\rho_1 A_1 (l_{11} + l_{12}) \omega_0}, \quad \omega_0^2 = \frac{E_{11} J_{11}}{\rho_1 A_1 (l_{11} + l_{12})^4}, \quad R_1 = R_2, \quad k_1 = k_2, \\
K = \frac{k_1 (l_{11} + l_{12})^3}{E_{11} J_{11}}, \quad \eta = \frac{E_{mn}}{c E_{mn}}, \quad c^2 = L_c \sum_{m,n} \rho_{mn} A_{mn} E_{mn} J_{mn} \sum_{m,n} E_{mn} J_{mn}
$$

The results of the calculations are presented in Figs. 2 to 5. Due to substantial differences in the values between real and imaginary parts of eigenvalues, the results were presented in different figures. A dependency of the real part $Re(\omega^*)$ and imaginary part $Im(\omega^*)$ of the first eigenvalue of the crane boom on total length $L_c$ and rigidity of internal damping $\eta$ is presented in Figs.
2a,b. Calculations were carried out for the coefficient of damping in slides \( \mu = 0.6 \). Fig. 3 illustrates the results of the investigations of the relationships between first eigenvalue of the crane boom and total length \( L_C \) and coefficients of damping in slides \( \mu \) at selected internal damping coefficient and spring rigidity. Fig. 4 illustrates the relationships between the first eigenvalue of the telescopic crane boom on simultaneous changes in internal damping coefficient \( \eta \) and constructional damping coefficient \( \mu \) with rigidity of springs \( K=5 \) and total crane boom length of \( L_C = 16.89 \text{[m]} \). The effect of changes in coefficients of damping in slides and spring rigidity on eigenvalues in the system for crane boom extension of \( L_C = 16.89 \text{[m]} \) and \( \eta = 0.002 \) is illustrated in Fig. 5a,b.

![Fig. 2](image1.png)

Fig. 2. The dependency of real parts (Re(\( \omega^* \))) and imaginary parts (Im(\( \omega^* \))) of the first eigenvalue on damping coefficient \( \eta \) and boom length \( L_C \) for \( \mu = 0.6 \) and \( K=5 \).

![Fig. 3](image2.png)

Fig. 3. The dependency of real parts (Re(\( \omega^* \))) and imaginary parts (Im(\( \omega^* \))) of the first eigenvalue on the coefficient of damping \( \mu \) and boom length \( L_C \) for \( \eta = 0.002 \) and \( K=5 \).
Influence of Damping in Slides on Vibrations of Telescopic Boom of Truck Crane

4. Conclusions

In this work a beam model of the telescopic crane boom system developed on the basis of the real system of the DST0285 truck crane has been presented. A study on the influence of the geometry, simultaneous internal damping of the viscoelastic material of beams modeling the system and constructional damping in slides between telescopic boom components into the system eigenvalues has been conducted. The study confirmed that taking damping in the system into consideration causes similar changes in the frequencies of damped vibrations with changes in system geometry as in the system without damping (2a, 3a). Substantial changes can be observed in the coefficient of amplitude decay $\text{Im}(\omega')$ in the case of changes in $Lc$ with changes in coefficient of internal damping (2b) and constructional damping (3b). Opportunities for determination of the length of extension of the crane boom for which the coefficient of amplitude decay is the highest allows for determination of the length of crane boom for which amplitudes
of system vibration will be minimal. The constructional damping in slides with selected value of internal vibration (4a,b) causes considerably higher changes in eigenvalues of the system than in the opposite case (change in the value of coefficient \( \eta \) with selected value of \( \mu \)). Changes in the spring rigidity \( K \) and damping in slides at selected extension of the boom crane and internal damping coefficient (5a) has the effect on the damped vibration which is analogous to the system without damping. The coefficient \( In(\omega^2) \) increases in the whole range studied with the increase in the coefficient of damping in slides whereas it decreases with the rise of rigidity of the springs which model slides (5b).

References


