Modelling and Analysis of Damping in Slides of Truck-Mounted Crane Boom

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Abstract

This study formulates and solves the problem of transverse damped vibrations in a crane boom of a truck-mounted crane. Dissipation of vibration energy in the model adopted results from motion resistance in slides between the moving components of the crane boom. A damping using Kelvin-Voigt model that characterizes dynamic properties of slides was added to the system. The effect of changes in damping coefficient and rigidity of springs on eigenvalues of the system studied was presented. The damped vibration frequencies and the degree of amplitude decay were calculated.

Keywords: vibration damping, crane boom, slides

1. Introduction

Analysis of vibration in a system for changing the boom radius in truck-mounted cranes has been carried out in numerous studies [Chin, Nayfeh and Abdel-Rahman 2001, Sochacki and Tomski 1999, Kilicaslan, Balkan and Ider 1999, Spruogis B., Jakštas A., Turla V., Iļjin I., Šešok N. 2011, Sochacki W. 2007]. The study [Posiadala and Cekus 2008] presented the vibration of the truck-mounted crane considered as a discrete system. The study used two-beam replacement model of a three-component crane boom. The effect of structural damping on transverse vibrations of the system of changing the boom radius was presented in the study [Sochacki and Bold 2013]. In the above mentioned study, dissipation of vibration energy was caused by the motion resistance in the points of cylinder fixation and crane fixation to the rotary frame of the crane, modelled as rotary viscous dampers. This study presents the effect of damping in slides between moving components of the crane boom on its transverse vibrations. In the replacement model studied, the actual three-component truck-mounted crane (DST0285) was modelled in the form of four beams (Bernoulli-Euler model) connected with elastic-damping systems (Kelvin-Voigt model) that represent dynamic properties of slides.
2. Physical and mathematical models of the system

The physical model of a telescopic crane boom radius is presented in Fig. 1. The model takes into consideration the elasticity of slides by modelling the slides with springs with elasticity constants \( k_1 = k_2 \) [N/m]. Damping in the slides of the crane boom was modelled using translational dampers denoted as \( R_1 = R_2 \) [Ns/m]. Actual crane boom in DST 0285 crane, composed of three components, was modelled with the beam system with lengths of individual beams of \( l_{ij} \) (\( i,j = 1,2 \)). The cylinder for changes in the crane was modelled with the spring with elasticity \( k_S \) [N/m].

![Fig. 1. Physical model of telescopic crane boom](image)

Equations of motion for individual beams in the model were denoted as:

\[
E_{mn} J_{mn} \frac{\partial^4 W_{mn}(x,t)}{\partial x^4} + \rho_{mn} A_{mn} \frac{\partial^2 W_{mn}(x,t)}{\partial t^2} = 0
\]  

(1)

where:
- \( W_{mn}(x,t) \) – transverse displacements of beams
- \( E_{mn} \) – Young's modulus for individual beams,
- \( A_{mn} \) – cross-sectional areas of the beams,
- \( J_{mn} \) – moment of inertia in beam cross-sections,
- \( \rho_{mn} \) – beam material densities
- \( m = 1,2 ; n = 1,2 \)
- \( x \) – spatial coordinate,
- \( t \) – time

Solutions for the equations (1) are given by:

\[
W_{mn}(x,t) = W_{mn}^* e^{i \omega_{*} t}
\]  

(2)

where: \( \omega^* \) – integrated eigenvalue of the system, \( \omega^* = Re(\omega^*) + iIm(\omega^*), i = -1^{1/2} \)
Substitution of (2) with (1) yields:

$$w_{mn}^{IV}(x) - \gamma_{mn} w_{mn}(x) = 0$$  \hspace{1cm} (3)

where:

$$\gamma_{mn} = \frac{\rho_{mn} A_{mn} \omega^{*2}}{E_{mn} J_{mn}}$$, \hspace{1cm} (4)

The geometrical and natural boundary conditions are as follows:

$$w_{11}(0) = 0, \quad E_{11} J_{11} w_{11}^{IV}(0) = 0, \quad w_{11}(l_{11}) = w_{12}(0), \quad w_{11}^{I}(l_{11}) = w_{12}^{I}(0),$$

$$E_{21} J_{21} w_{21}^{IV}(0) = 0, \quad E_{11} J_{11} w_{11}^{II}(l_{11}) = E_{12} J_{12} w_{12}^{II}(0), \quad E_{11} J_{11} w_{11}^{III}(l_{11}) +$$

$$-E_{12} J_{12} w_{12}^{III}(0) - (i R_{1} \omega^{*} + k_{1})[w_{11}(l_{11}) - w_{21}^{I}(0)] - k_{S} w_{11}(l_{11}) +$$

$$-E_{21} J_{21} w_{21}^{III}(0) = 0, \quad E_{12} J_{12} w_{12}^{II}(l_{12}) = 0, \quad w_{21}(l_{21}) = w_{22}(0),$$

$$w_{21}^{I}(l_{21}) = w_{22}^{I}(0), \quad E_{12} J_{12} w_{12}^{III}(l_{12}) - (i R_{2} \omega^{*} + k_{2})[w_{12}(l_{12}) - w_{22}^{I}(l_{12})] +$$

$$-E_{21} J_{21} w_{21}^{III}(l_{21}) = 0, \quad E_{21} J_{21} w_{21}^{II}(0) - (i R_{1} \omega^{*} + k_{1})[w_{21}^{I}(0) - w_{11}(l_{11})] +$$

$$+E_{12} J_{12} w_{12}^{III}(l_{22}) = 0, \quad E_{22} J_{22} w_{22}^{II}(l_{22}) = 0, \quad E_{21} J_{21} w_{21}^{III}(l_{21}) = E_{22} J_{22} w_{22}^{II}(0),$$

$$E_{21} J_{21} w_{21}^{III}(l_{21}) = E_{22} J_{22} w_{22}^{II}(0) - E_{22} J_{22} w_{22}^{III}(0) +$$

$$-(i R_{2} \omega^{*} + k_{2})[w_{21}(l_{21}) - w_{12}(l_{12})] - E_{12} J_{12} w_{12}^{III}(l_{12}) = 0.$$  \hspace{1cm} (5)

The solution for the equations (3) is given by:

$$w_{mn}(x) = C_{1mn} e^{\lambda_{mn} x} + C_{2mn} e^{-\lambda_{mn} x} + C_{3mn} e^{i \lambda_{mn} x} + C_{4mn} e^{-i \lambda_{mn} x}$$  \hspace{1cm} (6)

Substitution of (6) to (5) yields a homogeneous system of equations with respect to unknown constants $C_{fmn}$, which, in the matrix form, can be written as:

$$[A(\omega^{*})] C = 0$$  \hspace{1cm} (7)

where:

$$A(\omega^{*}) = [a_{pq}], \hspace{0.5cm} (p, q = 1, 2, 16), \hspace{1cm} C = [C_{fmn}]^{T}, \hspace{0.5cm} f = 1, 2 - 4$$  \hspace{1cm} (8)

The system has a non-trivial solution if the determinant of the coefficient matrix (with constant $C_{fmn}$) equals zero.

$$\det A(\omega^{*}) = 0$$ \hspace{1cm} (9)

Finding integrated eigenvalues of the matrix $A(\omega^{*})$ leads to determination of damped eigenvalues and the degree of amplitude decay in the system studied.
3. Results of calculations

Calculations were carried out for a telescopic truck-mounted crane boom (DST0285). A dimensionless damping coefficient $\mu$ and elasticity constant $K$ were adopted for presentation of the results.

$$\mu = \mu_1 = \mu_2 = \frac{R_1}{\rho_1 A_1 (l_{l1} + l_{l2}) \omega_0}, \quad \omega_0^2 = \frac{E_{11} J_{11}}{\rho_1 A_1 (l_{l1} + l_{l2})^3}, \quad R_1 = R_2.$$  

$$k_1 = k_2, \quad K = \frac{k_1 (l_{l1} + l_{l2})^3}{E_{11} J_{11}}$$  \hspace{1cm} (10)

The results of the calculations are presented in Figs. 2 to 6. Figure 2 illustrates the relationships between the first non-damped frequency of vibrations of the telescopic crane boom and changes in the length for two selected values of rigidity of springs $K$ (2a) and changes in rigidity of springs for selected total length of the crane boom (2b).

Fig. 2. The dependency of the first eigenvalue on length $L_c$ (a) and the dependency of the first eigenvalue on spring coefficient $k$ (b) for $\mu=0$.

Fig. 3. The dependency of the first eigenvalue on spring coefficient $k$ and length $L_c$ for $\mu=0$. 
The collective results (in the form of a spatial diagram) for the relationships between the first frequency of system vibration and changes in the rigidity of springs $K$ and total length of the crane boom $L_c$ without damping in the system are presented in Fig. 3.

The results of the examinations concerning the effect of damping in slides and their elasticity on vibrations of the crane boom are presented in Figs. 4 to 6. Due to substantial differences in the values between real and imaginary parts of eigenvalues, the results were presented in different figures. A dependency of the real part $Re(\omega^*)$ and imaginary part $Im(\omega^*)$ of the first eigenvalue of the crane boom on total length $L_c$ and rigidity of springs $K$ is presented in Figs. 4a,b. Calculations were carried out for the coefficient of damping in slides $\mu = 0.6$. Figure 5 illustrates the results of the investigations of the relationships between eigenvalues of the crane boom and total length $L_c$ and coefficients of damping in slides $\mu$ at selected spring rigidity. The effect of changes in coefficients of damping in slides and spring rigidity on eigenvalues in the system for crane boom extension of $L_c = 16.89$ [m] is illustrated in Fig. 6a,b.

![Fig. 4](image1.png)

*Fig. 4* The dependency of real parts ($Re(\omega^*)$) and imaginary parts ($Im(\omega^*)$) of the first eigenvalue on spring coefficient $k$ and length $L_c$ for $\mu = 0.6$.  

![Fig. 5](image2.png)

*Fig. 5* The dependency of real parts ($Re(\omega^*)$) and imaginary parts ($Im(\omega^*)$) of the first eigenvalue on the coefficient of damping $\mu$ and length $L_c$ for $k = 1.92168 \times 10^9$ [N/m].
4. Conclusions

Including damping during the investigations of vibrations of a crane boom with changes in its length and rigidity of the springs which model slides between its components causes only insignificant changes in the damped frequencies (4a) with respect to the system which did not include damping (3). Significant changes can be noticed in changes in the coefficient of amplitude decay $\text{Im}(\omega^*)$ in the case of changes in $L_c$ both with changes in spring rigidity (4b) and changes in the coefficient of damping in slides (5b). Opportunities for determination of the length of extension of the crane boom for which the coefficient of amplitude decay is the highest allows for determination of the length of crane boom for which amplitudes of system vibration will be minimal. Changes in the spring rigidity $K$ at selected extension of the boom crane and damping coefficient (6a) has the effect on the frequencies damped which is analogous to the system without damping (2b). The coefficient $\text{Im}(\omega^*)$ increases in the whole range studied with the increase in the coefficient of damping in slides whereas it decreases with the rise of rigidity of the springs which model slides (6b).

References


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