The Identification of the Elastic Support System of the Laboratory Truck Crane

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Abstract
In the paper the identification of the discrete-continuous model of the laboratory truck crane has been presented. In the theoretical model, the laboratory truck crane has been represented by three member type telescopic boom, hydraulic cylinder of crane radius change and the elastic support system. The free vibration problem of the analyzed system has been only considered in a rotary plane of the laboratory truck crane. For formulating and solving the free vibration problem of the analyzed system the Lagrange multiplier formalism has been employed. The identification of the discrete-continuous model has consisted in the determination of the spring constants substituting the elastic support system. Values of these spring constants have been determined on the basis of the solution of optimization problem and the experimental modal analysis. In optimization problem, the genetic algorithm has been used.

Keywords: identification, genetic algorithm, experimental modal analysis, hydraulic cylinder, telescopic boom, truck crane

1. Introduction
The construction of prototypes of all kinds of cranes or other labor-machines, which are subjected to experimental research, is impossible because of both economy and time. Therefore, modeling [Rusiński 2002, Eberhard and Schiehlen 1998] is applied. Modeling is the process of searching for the best representation of a real object that imitates behavior of a system in operating conditions. However, during the construction of models, the optimization of simplicity of a model as well as accuracy and faithfulness to a real object is necessary. If the achievement of sufficiently accurate results by using a less complex model is possible, then the model should not be extended. Discrete-continuous models are distinguished by this feature. However, in this case, a designer has to know all system parameters. Identification is useful in that. Identification [Ljung 1987, Uhl 2005] is a process of searching for model parameters for which the response of the model corresponds to the response of the real object. To identification of theoretical models the experimental studies are indispensable.
The telescopic boom is one of the most important elements of the truck crane and it has fundamental influence on the dynamic behavior of load and entire system during duty cycle [Sakazawa and Nakazumi 1985, Posiadała 1997, Kilicaslan et al. 1999, Sun and Kleeberger 2003, Sun and Liu 2006, Sochacki 2007]. Influence of the telescopic boom can be considered together with the other working structures of the truck crane. For instance, the hydraulic cylinder of crane radius change [Cekus and Posiadała 2002] is closely cooperating with the telescopic boom. The free vibrations of the system (built of the telescopic boom and the hydraulic cylinder of crane radius change) have been analyzed among other things in works [Posiadała and Cekus 2008, Cekus and Posiadała 2008, Cekus and Posiadała 2011], where vibrations in the rotary [Posiadała and Cekus 2008] and lifting planes [Cekus and Posiadała 2008, Cekus and Posiadała 2011] of the truck crane have been considered. However, in publications [Posiadała and Cekus 2008, Cekus and Posiadała 2008] the elastic support system has not been considered in models.

The identification of the theoretical vibration model of the laboratory truck crane in the rotary plane has been the aim of this work. The laboratory truck crane has been represented by the system three-member type telescopic boom – hydraulic cylinder of crane radius change and the elastic support system. The identification of the system consists in the determination of spring constants modeling the elastic reaction of the support. The solution of the optimization problem [Yang et al. 2007] and the experimental modal analysis [Bagheri and Jafari 2006, Karpel and Ricci 1997, Rusiński et al. 2008] have been used to identification. Genetic algorithm [Holland 1975, Goldberg 1989, Mitchell 1999] has been applied to optimization problem. For formulating and solving the vibration problem of the analyzed theoretical discrete-continuous model the Lagrange multiplier formalism has been applied [Posiadała 2007].

2. Theoretical vibration model of the analyzed system

The theoretical discrete-continuous model of the system three-member type telescopic boom – hydraulic cylinder of crane radius change in the rotary plane of the laboratory truck crane is shown in Fig. 1.

In this model the members of the telescopic boom are substituted by Bernoulli-Euler beams. The beams are connected by the springs $K_1$, $K_2$, $K_3$, and $K_4$ in the place where the slide blocks occur. The discrete-continuous model of transverse vibrations (described by coordinate $w_{2}(t)$ of the hydraulic cylinder of crane radius change) is joined to main member of the telescopic boom.

The cylinder and the piston rod of the hydraulic cylinder in the discrete-continuous model are also modeled by the Bernoulli-Euler beams. Elastic reaction acting between above elements is replaced by the springs $K_{c1}$ and $K_{c2}$. This reaction arises from influence of the piston and the seal. Whereas, influence of liquid filling the hydraulic cylinder has been restricted to the reaction of the inertial body of
mass $m_l$. The position and mass of this body undergo the changes during duty cycle of the laboratory truck crane.

Both the main member of the telescopic boom and the cylinder of the hydraulic cylinder are restrained with the help of the translational springs $K_b$ and $K_c$ as well as rotational springs $C_b$ and $C_c$. The applied systems of translational and rotational springs in the points, where the boom and hydraulic cylinder are attached to the platform, reproduce in the best way spring conditions of fixing real object. Moreover above systems of springs take into account reaction of the support system (base, chassis, four supports, and ground). Values of these springs have been identified.

![Fig. 1. The coupled system of analyzed model](image)

To formulate and solve the free vibration problem of the analyzed system the Lagrange multiplier formalism [Posiadała 2007], which is especially useful in cases of calculating the frequencies of combined systems that consist of many miscellaneous continuous and discrete elements, has been employed.

According to this formalism the description of the solution can be obtained in the following form:

$$
\bar{C} \bar{\lambda} = 0 ,
$$

where:

$$
\bar{\lambda} = \begin{bmatrix} \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8, \lambda_9, \lambda_{10}, \lambda_{11}, \lambda_{12} \end{bmatrix}^T
$$

is the vector of Lagrange multipliers and the square matrix $C$ has the form:

$$
C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}.
$$
Sub-matrixes $C_{11}$ and $C_{22}$ represent the transverse vibrations of the telescopic boom and the hydraulic cylinder of crane radius change respectively, and sub-matrixes $C_{12}$ and $C_{21}$ couple vibrations of both of element sets in the analyzed complex system. These sub-matrixes have the following form:

$$
\begin{align*}
C_{11} &= 
\begin{bmatrix}
C_{1,1} & C_{1,2} & C_{1,3} & C_{1,4} & C_{1,5} & 0 & 0 \\
C_{1,2} & C_{1,3} & C_{1,4} & C_{1,5} & 0 & 0 \\
C_{1,3} & C_{1,4} & C_{1,5} & 0 & 0 \\
C_{1,4} & C_{1,5} & 0 \\
0 & 0 & 0 & -C_{5,6} & -C_{5,7} & -C_{5,8} & -C_{5,9} \\
0 & 0 & 0 & -C_{5,6} & -C_{5,7} & -C_{5,8} & -C_{5,9}
\end{bmatrix}, \\
C_{12} = C_{12}^T &= 
\begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix}, \\
C_{22} &= 
\begin{bmatrix}
C_{4,11} + \varepsilon_{6} & C_{4,21} & C_{4,31} & C_{4,11} & C_{4,12} \\
C_{4,11} & C_{4,22} + \varepsilon_{7} & C_{4,32} & C_{4,12} & C_{4,13} \\
C_{4,12} & C_{4,23} & C_{4,33} + \varepsilon_{8} & C_{4,13} & C_{4,14} \\
C_{4,12} & C_{4,24} & C_{4,34} + \varepsilon_{9} & C_{4,14} & C_{4,15} \\
C_{4,13} & C_{4,25} & C_{4,35} & C_{4,15} & C_{4,16} \\
C_{4,13} & C_{4,26} & C_{4,36} + \varepsilon_{10} & C_{4,16} & C_{4,17} \\
C_{4,14} & C_{4,27} & C_{4,37} + \varepsilon_{11} & C_{4,17} & C_{4,18} \\
C_{4,14} & C_{4,28} & C_{4,38} + \varepsilon_{12}
\end{bmatrix}.
\end{align*}
$$

(4a)

The subsequent denotations have been introduced in the matrixes (4):

$$
C_{n,i} = \sum_{i=0}^{N} b_{n,i} b_{n,i},
$$

(5)

$$
\varepsilon_1 = 1/K_1, \ varepsilon_2 = 1/C_b, \ varepsilon_4 = 1/K_1, \ varepsilon_5 = 1/K_2, \ varepsilon_6 = 1/K_1, \ varepsilon_7 = 1/K_4.
$$

(6a-f)
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The coefficients \( C_{nk,r} \) defined by the formula (5) characterize dynamic properties of beams representing the members of the telescopic boom \((n=1,2,3)\), the cylinder \((n=4)\) and the piston rod \((n=5)\) of the hydraulic cylinder.

The quantities \((\varepsilon_k)\) described by the relationships (6) define the influence of discrete elements on the vibrations of the system and represent: the elasticity of the slide blocks occurring between the members of the telescopic boom \((\varepsilon_4, \varepsilon_5, \varepsilon_6, \varepsilon_7)\); the inertial reaction of liquid filling the cylinder of the hydraulic cylinder \((\varepsilon_{10})\), the elasticity of connection between the cylinder and the piston rod \((\varepsilon_{11}, \varepsilon_{12})\) in the hydraulic cylinder and the elastic reaction of support system \((\varepsilon_1, \varepsilon_2, \varepsilon_8, \varepsilon_9)\).

From the condition of existing of nontrivial solution of system of equations (1) the equation of free vibration frequencies of the system has been obtained in the form of:

\[
\det C = 0. \tag{7}
\]

To complete the description of the theoretical discrete-continuous model some mathematical expressions from above mentioned work [Posiadała 2007] are necessary to add.

On the basis of relationship (7), the free vibration frequencies \(\omega_k\) have been determined and then corresponding modes have been described. It is vital to add that the determining of modes be possible after solving Eq. (1) in relation to the Lagrange multipliers (separately for each vibration frequency \(\omega_k\), which the mode matches). On account that the system of Eq. (1) is homogeneous, the choice of one amplitude as independent parameter is necessary and the rest of the amplitudes can then be determined in the relation to that arbitrary chosen parameter.

2. The experimental research

The experimental research has been carried out on the laboratory truck crane [Tomski and Chwalba 1989]. This model has been built to represent the real truck crane in the geometrical scale 1:5 and is schematically presented in Fig. 2a.

It consists of: three-member type telescopic boom (1) and the hydraulic cylinder of crane radius change (2), which are attached to the base (3). The base is rotationally mounted in chassis (4) supported on the four supports (5). All motions of the laboratory truck crane are realized by hydraulic systems which ensure the realization of movements of two telescopic boom members, rotation and slope of the telescopic boom.

The measurement system used in the research is presented in Fig. 2b and it consists of: the PC computer (6) with PULSE and ME’scopeVES programs; instrumentation of Brüel&Kjær: four-channel vibration analyzer 3560C (7),
amplifier 2707 (8), exciter body 4805 with exciter head 4811 (9), force detector (10), one-axial piezoelectric accelerometer (11).

On the basis of carried out experimental modal analysis (only in the rotary plane of the laboratory truck crane), the set of natural frequencies and modes of vibrations of telescopic boom and hydraulic cylinder of crane radius change have been obtained.

In Fig. 3, first four values of free vibration frequencies of the system and corresponding with them modes (top view and isometric view) are compiled.

![Fig. 2. Scheme of the laboratory truck crane (a) and the measurement system to experimental modal analysis (b)](image)

3. The identification of the elastic support of the analyzed system

Using derived relationships and results of experimental research, the identification of springs’ constants modeling the elastic influence of the support system of the laboratory truck crane has been carried out. Values of these springs’ constants have been determined on the basis of formulation of optimization problem. The genetic algorithm has been the optimization algorithm.

In the optimization process the following object function has been accepted:

\[
f(K_b, C_b, K_e, C_e) = \frac{\sum_{i=1}^{k} |\omega_i^{(0)} - \omega_i^{(e)}|}{\omega_i^{(e)}} \times 100\% \quad \text{for} \quad k = 4,
\]  

(8)
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which means that the average relative error between the first three values of free vibration frequencies from experimental research \( (\omega_e) \) and theoretical calculations \( (\omega_t) \) has been optimized.

The worked out algorithm and numerical program enable obtaining optimum values of spring constants \( K_b, C_b, K_c \) and \( C_c \). Computations have been carried out by the following data:

- the analyzed system parameters (the same as during the experimental research): the whole length of the telescopic boom has been equal 5.2 m, at the same time each moving member of the telescopic boom has been pulled out on the 1.5 m and the height of the column of the fluid filling the cylinder has been equal 0.1 m,

- the genetic algorithm parameters: the crossover probability – 80%, the mutation probability – 10% and the rank selection has been used,

- the values of springs constants \( (K_b, C_b, K_c \) and \( C_c) \) have been looked for in the range \( [1000, 1\cdot10^10] \).

From the numerical calculations:

- the average relative error Eq. (8): 0.013%,

- the values of constant springs representing the support system:
  \[ K_b=157379 \text{ N/m}, \quad C_b=60197.7 \text{ Nm}, \quad K_c=28913.4 \text{ N/m}, \quad C_c=29926.8 \text{ Nm} \]

have been obtained.

In Fig. 4, the values of free vibration frequencies and corresponding modes of the telescopic boom (a), the hydraulic cylinder of crane radius change (b) received on the basis of numerical calculations have been shown. The numerical research has been conducted with the use of determined values of spring constants modeling the support system. The symbols \( W(k) \) and \( W_{c2}(k) \) \( (k=1,2,3,4) \) denote the motion amplitudes of the end of the telescopic boom and displacement of the end of the hydraulic cylinder, where the number \( k \) denotes the mode number.

Comparing the experimental (Fig. 3) and the calculated (Fig. 4) free vibration frequencies and the modes, one can notice compatibility within the values and the next modes as for the shape and the number of nodes for the individual elements of the system that are the telescopic boom and the hydraulic cylinder. It allows to state that the accepted theoretical model by the calculated values of spring constants representing the support system expresses the real object (the laboratory truck crane) appropriately.
Fig. 3. The experimental free vibration frequencies and the modes of the analyzed system obtained on the basis of experimental modal analysis: a) top view, b) isometric view
Fig. 4. The numerical free vibration frequencies and the modes (for the system with elastic support) of: a) the telescopic boom, b) the hydraulic cylinder of crane radius change

4. Final remarks

In the paper, with the help of genetic algorithm and experimental modal analysis, the discrete-continuous model of the system three member type telescopic boom – hydraulic cylinder of crane radius change has been identified. Only vibrations in the rotary plane of the laboratory truck crane have been considered.

The identification of this model, which has been formulated and solved according to the Lagrange multiplier formalism, consisted in determining values of translational and rotational springs (modeling the support system) constants.
On the basis of comparison of experimental and theoretical modal models can state that the accepted identified theoretical model illustrates the laboratory truck crane appropriately. It allows conducting the analyses of the influence of the hydraulic cylinder and the whole length of the telescopic boom on the free vibration frequencies and modes.

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References


