Nonlinear Dynamics and Control of an Active Suspension of a Pendulum Vibration Absorber

Krzysztof Kęcik, Andrzej Mitura
Department of Applied Mechanics, Lublin University of Technology (LUT)
e-mail: k.kecik@pollub.pl, a.mitura@pollub.pl

Abstract

The paper presents concept of an active suspension of an autoparametric pendulum system. The suspension consist of a magnetorheological (MR) damper and a shape memory alloy spring (SMA). The influence of non-linear damping and SMA stiffness on regular and chaotic vibrations and their stability, near the main parametric resonance has been analyzed. This active suspension allows control of position of periodic or chaotic regions. Additionally, the proposed suspension does not reduce the absorption effect.

Keywords: pendulum, MR damper, SMA spring, chaos, stability, stability, control.

1. Introduction

In vibration theory, dynamic vibration absorber is a system which reduces or eliminates the dangerous vibration. This type of devices can be control by semic-active method, which is a compromise between passive and active control. The controlling effect is applied by including within the semi-active element a physical process which acts as though a system parameter is being varied. The study a dynamic vibration absorbers has been gradually increasing. One of DVA example is an autoparametric pendulum vibration absorber (APVA) [Haxton R.S., 1972], which is probably the earliest passive device that makes use of a purely nonlinear response for vibration suppression [Cartmell M.P., 1990]. The APVAs have an interesting dynamics, due to at least two nonlinearly coupled subsystems (called main and absorber) interacting each other to transfer the exogenous perturbation energy into an absorber. In most dynamics applications, the DVA is tuned so that its natural frequency is close a problem resonance frequency of the original system. Additionally, the response of the DVA has a lower amplitude compared to that of the original system in the vicinity of the problem frequency. Properly designed and tuned absorbers reduce vibration selectively in the maximal vibrations mode without inducing vibration in other modes [Kecik K., et al., 2014].
In order to determine the optimal systems parameters for a pendulum vibration absorber complete analysis of its dynamics is necessary.

This paper presents an autoparametric pendulum vibration absorber suspended on a novel active suspension, which consist of a magnetorheological damper (MR) and spring made from a shape memory alloy (SMA). The MR damper used for controlling damping of system, but SMA spring is used to change the stiffness. Additionally, influence of MR and SMA elements on chaos and periodic solutions and stability is presented also.

2. Modelling of the an active system

2.1 Mechanical model

The system under study consists of an oscillating mass and a vibration absorber (pendulum), as shown in Fig. 1. The motions of the system is described by two generalized coordinates, namely the displacement of the oscillator in the vertical direction (X), and the angular displacement of the pendulum (\( \phi \)) measured in anti-clockwise direction. The system is excited by harmonic force \( F(\tau) \) in vertical direction. The damping in the pendulum pivot is assumed as viscous, described by coefficient \( \alpha_2 \).

![Fig. 1. Model of an active autoparametric pendulum vibration absorber.](image)

The dimensionless equations of motion for the two degrees of freedom autoparametric vibration absorber, using Lagrange approach, has been obtained as follows:

\[
\ddot{X} + F_{MR} \left( \dot{X}, X \right) + F_{SMA} \left( \theta, T \right) + \mu \left( \dot{\phi} \sin \phi + \dot{\phi}^2 \cos \phi \right) = q \cos \left( \theta \tau \right),
\]  

(1)
\[ \ddot{\varphi} + \alpha_2 \dot{\varphi} + \lambda (\ddot{X} + 1) = 0, \]  
(2)

where: \( q \) and \( \vartheta \) are amplitude and frequency of excitation, respectively. The parameters \( \lambda \) and \( \mu \) characterize the pendulum construction. The parts \( F_{\text{SMA}} \) and \( F_{\text{MR}} \) denote force in smart elements described in next paragraph. Note, that eqs. (1) and (2) are typical for an autoparametric system which included inertial term and quadratic nonlinearity. The detailed derivation of equations of motion is presented by author in paper [Warminski J., Kecik K., 2012].

### 2.2 An active element’s suspension

The proposed suspension of APVA consists of two elements with variable characteristics: MR damper and SMA spring, vertically mounted between the main system and background. To describe of MR damper, the model consist of viscous damping (\( \alpha_1 \)) and dry friction (\( \alpha_3 \)) is used

\[ F_{\text{MR}} (X, X) = \alpha_1 \dot{X} + \alpha_3 \tanh (\delta_1 \dot{X} + \delta_2 X), \]  
(3)

where: parameters \( \delta_1 \) and \( \delta_2 \) describes of slope dry friction characteristics and hysteretic loop, respectively. The detailed study of this model can be found in [Kecik K., et al., 2014].

One of the most popular constitutive models for describing the behaviour of SMA alloys comes from the original Landau-Ginzburg theory. In this approach, constitutive information is provided by a polynomial free-energy, function whose partial derivatives provide constitutive equations for strain (\( \varepsilon \)) and entropy. The behaviour of the SMA spring is described by the polynomial constitutive model, where the restoring force is determined by substituting (\( \varepsilon \)) by the displacement of the oscillator mass \( X \). In dimensionless form, the SMA force yield

\[ F_{\text{SMA}} (\theta, T) = (\theta - 1)X - \beta_1 X^3 + \beta_2 X^5, \]  
(4)

where: parameters \( \beta_1, \beta_2 \) characterize shape memory material, \( \theta \) is temperature ratio, between the activate and martensite phase temperature. Note, that for \( \theta = 2 \) and \( \beta_1 = 0, \beta_2 = 0 \) polynomial model of SMA spring became a linear. The model described by eq. (4) is commonly used, e.g. in papers [Sado D., et al., 2014, Sado D., Pietrzakowski M., 2010].

### 3. Numerical studies

#### 3.1 Influence of variable stiffness and damping on periodic motion and stability

The variable MR or SMA force (or both together) gives possibility to control of system. However to get the desired response, it is necessary to know the
influence of an active elements when the system works in regular and chaotic zones. The first, the influence of MR damping on periodic motion and stability is investigated. The control by this parameter is easy and quick to implementation in real system.

The resonance curves, by continuation technique are obtained, using Auto07p software. The resonance curves for the oscillator and the pendulum for a system with different MR damping, and classical spring ($\theta=2$, $\beta_1=\beta_2=0$) are shown in Fig.4. The case of response for classical viscous damping (then $\alpha_3=0$) is marked by red line. The light gray and dark gray lines correspond to the MR damped system, equals $\alpha_3=0.1$ and $\alpha_3=0.3$, respectively. The unstable solutions are marked by dash-dotted line, while the solid line denotes stable solutions. Close, to frequency $\theta\in(0.6-0.8)$, absorption effect is visible, but for $\theta=0.65$, the amplitude of the oscillator is lowest (smaller than the amplitude of excitation). Analysing, the results in both diagrams we can conclude that increase of MR damping will not eliminate of absorption effect (Fig.2a), but slightly reduces its width of the right side (clearly visible in the pendulum region swinging in Fig. 2b). Of course, the amplitude of the oscillator for semi-trivial solution (case of response with a fixed, non oscillating pendulum) significantly reduced. The point at the tip of the parabolic curve is labelled L (limit point), meaning that it represents a fold bifurcation. The point D denotes a period-doubling bifurcation point, while point B is a branch point, which is just a fancy way of saying that a bifurcation occurs here. The point T denotes Neimark-Sacker (torus) bifurcation, in this case an invariant torus bifurcates from the periodic solution.

An alternative way to control the absorber dynamics is to change of stiffness in suspension point. This may be realized by suitable construction of a spring or by application of spring made of shape memory alloy. The temperature ratio $\theta$ of SMA spring has significant influence on the dynamics and stability.

![Fig. 2](image_url)

Fig. 2. Frequency response for the oscillator (a) and the pendulum (b), for system with a MR damper: $\alpha_1=0.26$, $\alpha_2=0.02$, $\mu=17.2$, $\lambda=0.12$, $q=0.5$, $\delta_1=10$, $\delta_2=1$, $\theta=2$, $\beta_1=0$ and $\beta_2=0$. 
This effect in Fig. 3a and Fig 3b is shown, where light gray colour identifies $\theta=2$, dark gray $\theta=1.5$ and black $\theta=1$. The increase of temperature SMA element, can change unstable in stable solutions, and significantly reduces number of solutions. Note, for $\theta=1$, a lot unstable solutions are possible (practically most unstable, see Fig. 3a). Only, for $\theta=0.6$, the periodic stable semi-trivial solution is possible. Additionally, the increase in $\theta$, can improve the absorption effect (but reduces frequency resonance region), what is important from practical point of view.

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Fig. 3. Frequency response for the oscillator (a) and the pendulum (b), for system with SMA spring: $\alpha_1=0.26$, $\alpha_2=0.02$, $\mu=17.2$, $\lambda=0.12$, $\eta=0.5$, $\delta_1=10$, $\delta_2=1$, $\beta_1=0.52$ and $\beta_2=0.26$.

The use of SMA spring influence on numbers and new bifurcation points (Neimark-Sacker bifurcation appears). Additionally, the semi trivial solution (only oscillator vibrates), can be unstable. Comparing of influence of MR damping and SMA spring we can conclude, that SMA spring radically change the response of the system, and introduce a new solutions.

3.2 Influence on chaos

The control of dynamics by an active suspension is possible, but different solutions are possible. From, the absorption point of view, particularly chaos region is most dangerous. Therefore, the influence of MR damping and SMA spring stiffness parameters on irregular motion are analysed.

The chaotic regions under MR damping influence in two parameter space plot are presented in Fig.4a. The amplitude of excitation was increased, comparing to results presented in Fig.2. The calculation are performed in such a way that the first 200 solution periods are excluded. The calculation are done in Dynamics 2 software, with Runge-Kutta integration method [Nusse H.E., Yorke J.A., 1998]. For each value of the varied parameter the same initial conditions, $\psi(0)=0.1$, $d\psi/dt(0)=dx/dt(0)=x(0)=0$ are taken. The dark blue, cyan and green colours
indicate chaotic regions estimated on the basis of positive value of the maximal Lyapunov exponent. White colour defines periodic motion, oscillation, rotation or regions where the pendulum goes to the lower equilibrium state. As we may see, chaotic response occurs near the frequency $\omega \approx 0.55-0.7$ and $\omega \approx 1-1.3$. Introducing MR damping we reduced chaotic tongues and parts of chaotic areas are divided into smaller domains.

![Fig. 4. Influence of MR damper ($\alpha_3$) for $q=2$, $\beta_1=\beta_2=0$, $\theta=2$ (a) and SMA spring, ($\theta$), for $q=0.5$, $\beta_1=0.52$, $\beta_2=0.26$, $\alpha_3=0$ (b) on chaotic behaviour.](image)

The change of temperature ratio of SMA spring can may lead to chaotic behaviour, even for small value of excitation ($q$). These results in Fig. 4b are shown, where blue colour indicates chaotic behaviour, located near $\omega \approx 0.6-0.9$. Note, that results of Fig. 4b are similar to the resonance curves obtained by continuation technique (Fig. 3b). But, this diagram shows chaotic motion for nontrivial solution (the pendulum and oscillator swinging), for one set of initial conditions, only. Interestingly, that chaotic region is located near the absorption region. Reduction of temperature in this case change stability from stable in unstable and chaotic motion appears (see Fig.3a, $\theta=2$, $\theta=1.5$ for $\omega \approx 0.6$).

4. Conclusions

This paper delivers a vibration analysis of a nonlinear pendulum absorber suspended on an active suspension consists of MR damper and SMA spring. The MR damper is modeled with hysteretic effect, while SMA spring is modeled by constitutive polynomial model. The MR damping ($\alpha_3$) generally eliminates chaotic motion and does not reduce the absorption effect. This result is essential, because can be used to control the system without a loss of an efficiency of dynamic vibration suppression.
The SMA spring rapidly can change dynamics and introduce a new solutions (most unstable) and bifurcations (Neimark-Sacker). Additionally, the chaotic motion can appears, especially for lower value of $\theta$ and near the absorption region. The smarts elements can be used to control, however detailed analysis based on basins of attractors (especially for SMA spring), before control application should be made.

The next step of the investigations is apply a closed-loop control in real system, and energy harvester in such systems.

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References


