Construction Methods of Pulley Model with Non-Circular Rolling Line for the Cax Analysis

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Abstract
The paper presents the construction procedure of pulley models with the noncircular rolling line which are applied in uneven-running strand transmissions with the cogbelt. The characteristic feature of these type of transmissions is the cyclic variation of kinematic and dynamic features. The construction of a CAD model is the basic task in designing process and analysis of the transmission. The CAD model is applied for the elaboration of the machine programs for the manufacturing with the use of CNC, measuring the geometrical features with the use of optical and contact methods and also for the finite element analysis of the transmission.

Keywords: cad modeling, manufacturing, measurement, FEM

1. Introduction

Manufacturing of cogbelts with a noncircular pitch line and variable values of pitches on the circumference with the use of classical manufacturing methods (enveloped methods) is difficult to perform. The design of variable values of pitches is a condition of the proper engagement of cogbelt (with constant pitch) and noncircular cogbelt pulley (with noncircular profile). In case of circular cogbelt pulleys this problem does not exist. If the noncircular cogbelt pulley is made with a constant value of pitch then the cogbelt will not engage with this pulley. Manufacturing of noncircular cogbelt pulleys with the use of NC milling machine, gear-shaper cutter, a set of end mills, profile cutter, wire-cut electric discharge machine, water cutting with abrasive, laser cutting, metal powder sintering were examined. Mating correctness of a noncircular cogbelt pulley and a cogbelt must be verified before beginning of production (before manufacture of tool, instrumentation and elaboration of manufacturing process) in order to avoid negative and costly problems such as manufacture of incorrect profile cutter, gear-shaper cutter

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or geometrical and stereometrical features which are inconsistent with standard requirements (in case of water cutting with abrasive). The purpose of research was to check the correctness of the designed CAD model which was a basis for the analysis of manufacturing methods. Therefore, the results of research are very useful because these ones restrict the errors formation, e.g. wrongly designed tool, application of unnecessary and expensive mechanical working. The required rate of unevenness is obtained with the application of cogbelt pulleys which have elliptical, oval or noncircular disc profiles on the circumference of the rim. The application of chains in these transmissions is already known. However, the problem of cogbelt application in these transmissions has not been recognised yet. The correctness of designing the teeth of noncircular cogbelt pulleys has to be verified both with simulation methods and physical models. This will allow one to verify the correctness of the shape-frictional conjugation between the noncircular cogbelt pulley and the timing belt. The forming methods of geometrical features and surface stereometry for noncircular cogbelt pulleys were presented in the works of [Krawiec P., 2009, Kujawski M. and Krawiec P., 2011]. Production of a single number of noncircular cogbelt pulleys with machining methods is connected with the need to elaborate the non-typical manufacturing process and the application of unique tools or materials.

2. CAD modelling

The process of shape-frictional conjugation in a transmission with noncircular cogbelt pulleys is quite different from the same process in a transmission with traditional belt pulleys. The difference between these conjugation processes results directly from the geometrical features of the cogbelt pulleys, which can be elliptical or oval, as well as from their kinematic features. The basic problem in the conjugation characteristics for this type of transmission is the need to design variable pitches in noncircular cogbelt pulleys. This need should be satisfied so as to obtain a proper engagement of the noncircular cogbelt pulley and the standard cogbelt. The engagement requires an adjustment of the “constant” pitch of the cogbelt to the temporary engagement conditions in a transmission. The problem of variation of pitches has to be taken into account during the geometry of cogbelt pulleys with a variable value of pitches can be done with classical machining methods, but this is a difficult process. The application of, for example, one cutting tool with the same profile for the whole noncircular cogbelt pulley results in the tooth space becoming damaged or not being machined at all. This problem is illustrated in Fig.1.
Fig. 1. Errors of a noncircular (elliptical) cogbelt pulley which was machined with a profile cutter

The positions of the centres and values of the curvature radii for the noncircular cogbelt pulley are presented in Fig. 2.

Fig. 2. Geometry of the noncircular (elliptical) cogbelt pulley: R1-R12 - parts of an envelope of the noncircular cogbelt pulley with a variable pitch \( p_z \); \( r_z \) - outer radius of the noncircular cogbelt pulley, \( a \) - distance between outer radius \( r_z \) and pitch radius \( r_p \)

These parameters can be evaluated from the generally known relationships for curvature of a plane curve. Fig. 2 presents a part of the cogbelt pulley with a noncircular pitch line. This figure shows a variation of pitch values which were
described in first paragraph. In case of gears which are used in transmissions, the pitch radius is smaller than outer radius, while in cogbelt pulleys the pitch radius is greater than outer radius. Pitch values are calculated for variable values of outer and pitch radii. If the cogbelt pulley was circular, then the pitch would be equal to a constant value of 9.525 (for L-type cogbelt).

Table in Fig. 2 presents the calculated values of variable \( p_z \) and averaged \( p_s \) pitches. The profiles of noncircular cogbelt pulleys with variable values of pitches on the external profile were prepared in AutoCAD software. Next the obtained files were imported into Autodesk Inventor software and solid models were built.

### 3. Methods of forming pulleys

Like the circular wheel machining techniques also methods for manufacturing noncircular wheels are divided into:

- machining, in which we can distinguish profiling and envelope method and also point machining method, formerly known as the copy; the traditional method of copying was to transfer the outline (profile) of the pattern wheel on the wheel machined using ganging; at present in CNC transferring information about the geometrical features of wheel is on the way by the numerical control of mechatronic classical tools and this principle is used in both the end milling process and in the machining on EDM, where the tool is properly sized wire (WEDM - wire electrical discharge machining) and the same solution also applies on CNC contour band, where the tools are gas, water jet with abrasive or plasma;

- metal working;

- metal powder sintering technology.

In the case of using profile tool there is formed outline of the tooth without the profile movement as a reflection of the tool cutting edge profile. The use of profile tools is indicated for the high volume machining. During machining wheels with envelope tools there is formed involute tooth outline as an envelope of other positions of the tool during rolling without a rolling lines sliding of the tool and the workpiece. Just as in the manufacture of circular outline of the wheel pulley its outline tooth space corresponds to the outline of tool. In our example, the tool has been specially designed and manufactured profile cutter (Fig. 3). Its shape is designed so that during the processing of one wheel tooth space was machined at the same time the two halves of heads of adjacent teeth. Traditionally, this method was used in the process of machining on milling machine with dividing equipment by which obtained the appropriate division of wheel. At present, more accurate and simpler solution is to use numerically controlled milling machine with rotary table.
Due to the limitations of profiling method and taking into account that the machining of noncircular wheel envelope requires design of untypical technological process and application of numerical control of vertical shaper, whose popularity on the Polish market is small, so there has been attempted to develop own processing method of machining that combines the features of envelope shaping, and also profiling (Fig. 4).

Searching accurate and cheap methods of noncircular pulley manufacturing there has been taken an attempt of using for this an universal technological machine tools, such as: numerical control milling machines, wire contour band, water contour band, etc. and also advanced CAD and CAD/CAM systems. Moreover, there has been noted the need to use conventional tools and to ensure the machining
accuracy of parts in range of geometrical features and stereometrical surface. The method that satisfies these requirements with regard to noncircular pulleys may be machining of teeth on universal numerical milling machine (Fig. 5).

Fig. 5. Noncircular wheel made of end mill

The analysis of the literature on laser cutting has shown that justifies will be made of noncircular wheel of constructional steel and acid resistant steel. There has been assumed that the criterion of suitability of various methods of manufacture of wheels is possibility to avoid finishing machining. The first analyzed method of noncircular wheel forming was a gas laser jet cutting (Fig. 6).

On the manufacturing accuracy of the elements of this method have influence mainly two factors: properly maintained jet cutting and resistance to vibrations of the CNC machine tool. An important advantage of this machining method is repeatability of geometrical features and stereometrical surfaces of obtained products. If the parameters of technological process of machining parts using a laser beam will be properly selected, this method will become an alternative solution to some of the methods of machining. A major constraint influencing on the quality of this process is the thickness of the formed elements. In Fig. 6a and 6b are shown respectively pulleys made of carbon steel and acid resistant steel.
Another profiling method of the wheel teeth, shown in Fig. 7, were WEDM – wire electrical discharge machining. The tool of this machining was thin brass wire with a diameter 0.02–0.5 mm. There are commonly using wires made of copper, tungsten, molybdenum or wire with coating, such as galvanized brass. Cutting technology on CNC spark erosion machine is quite expensive in comparison to laser cutting or water jet cutting.

Application of laser cutting gas is greatly reduced due to the occurring of heat, and consequently deformation of the workpiece. Disadvantages of this machining are used of water jet cutting (Fig. 8).
This method can also be an alternative to conventional machining methods, requiring a design of technological complex process, and special tools, such as cams and gears with noncircular envelope rim, used in hydraulic machines.

One of the leading centers for the manufacture of machine parts of sintered metal powders is the Metal Forming Institute in Poznan, made possible the development of design and technological manufacture assumptions used in the noncircular wheel gear used in unevenness cogbelt pulleys made of sintered metal powders [Krawiec 2008].

Looking for a fast methods to estimate cooperation of the teeth belt and pulleys it is necessary made an attempt to use for forming noncircular wheel pulleys some techniques such as rapid manufacturing (RM) and rapid prototyping (RP). Among methods to rapid prototyping for manufacturing examples of wheels there has been selected FDM technique (Fig. 9a) – fused deposition modeling, and 3D printing (Fig. 9b).
4. Measurements of geometrical and surface stereometry of non-circular belt pulleys

The advantage of using manufacturing systems is obtaining a proper amount of products with repeatable geometrical and stereometrical features in a short time. Thanks to the application of measurement method of Rapid Inspection one can speed up the processing in a comparison with application of coordinate measuring techniques. Rapid Inspection as a measurement method is a significant link in Rapid Technology process. It allows to estimate quickly the results of measurement and makes easier to select the forming method beneficial influence on formation of repeatable errors of products and can be helpful in forming of optimal constructions. The width of the noncircular cogbelt pulleys was equal to 15±0.1 mm. The noncircular cogbelt pulleys which had been manufactured with the use of different methods were checked according to the following elaborated criteria: representation correctness of the designed geometrical features and surface stereometry, preparation complexity of the manufacturing process and production costs of a specified lot of noncircular cogbelt pulleys. To perform this analysis a procedure of experimental verification of the manufacturing correctness of noncircular cogbelt pulleys was elaborated. The author found that there are no requirements concerning standardisation of representation accuracy for the geometrical features and surface stereometry of noncircular gear wheels and noncircular cogbelt pulleys. Currently, the standards define the manufacturing accuracy of circular cogbelt pulleys only. Measurements of external profile of exemplary belt pulleys (Fig. 10), which were manufactured with application of two cutting edges tool, were conducted in three parallel planes which are perpendicular to belt pulley’s rotational axis.

Fig. 10. Measurement of geometrical features of noncircular belt pulley on coordinate measuring machine of Zeiss Contura company
Coordinate measuring machine of Zeiss Contra company was used for measurements of geometrical features. Measuring process of geometrical features was based on so-called contour searching method. Measurements of belt pulley’s stereometry were conducted with application of profilometer of Taylor Hobson company (Fig. 11).

The Direct action in the field of manufacturing and measurement should take into consideration two directions of research: the first direction should concern an elaboration of the standards which will describe new teeth profiles for cogbelt pulleys (involute, semicircular profile); the second one should concern the characteristic of requirements in the scope of manufacturing accuracy of noncircular cogbelt pulleys. The allowable values for the limiting deviations of the outer diameter and pitch errors for circular cogbelt pulleys according to the ISO 5204 [PN-84/M85211] standard.

During measurement of the geometrical features, two methods were applied, i.e. coordinate and optical methods. The second method is a measuring-and-controlling procedure called Rapid Inspection. This method consists in a comparison of the designed CAD model with the surface model of the formed pieces or units that have been created as a result of digitisation. The existing differences between these two models can easily be illustrated on a coloured map of deviations (Fig. 12).

One should remember that this method is burdened with a methodical error which results both from scanning accuracy and matching correctness of the models. This error is especially important during the analysis of complex models, e.g. for elements formed during plastic working processes.
The representation accuracy of geometrical features of products manufactured with any forming technologies (machining, plastic working, incremental methods, etc.) can be evaluated with the optical method.

5. FEM analysis

The examined numerical model of a strand transmission was elaborated in the Abaqus program. The model was described as a two-dimensional state of stresses. Geometrical models (solid models) of pulleys and cogbelt were prepared in AutoCAD and Autodesk Inventor programs. The transmission elements were transferred to the Abaqus program in DXF files. The initial configuration of the transmission – before assembly – is presented in Fig. 13. The pulleys and the cogbelt were discretized by 4-node and 3-node finite elements for a plane state of stresses with reduced integration and linear shape functions (CPS4R and CPS3 according to Abaqus denotation).

The cogbelt reinforcement was defined as 2-node bar element with linear shape functions (T2D2 according to Abaqus denotation). The bar elements were defined on the nodes of elements of plane state of stresses. The FEM model of the transmission (Fig. 14) consisted of 18 084 elements and 18 710 nodes and had 37 418 degrees of freedom.

The FEM model of the cogbelt had a composite structure (two materials) – Fig. 13. The rubber, which had been used as a body of the cogbelt, was described as isotropic and nonlinear-elastic material.
The speed of a vertical displacement and a rotation on axis were excited as zero. The simulation of this process lasted 2 seconds.

![Diagram of cogbelt](image)

**Fig. 13.** Composite structure (two materials) of the cogbelt

**Fig. 14.** Kinematic excitation (horizontal displacement – brown arrows) which stretches the cogbelt

**Fig. 15.** Energy balance for the first step (cogbelt stretching)

The diagram of the energy balance for the first step confirms the quasi-static character of the process (Fig. 15).
5. Conclusion

In paper there was researched various manufacturing methods in order to find an accurate, simple and cheap method of manufacturing noncircular cogbelt pulleys. A characteristic feature of noncircular cogbelt pulleys is the pitch variation on the external. The noncircular cogbelt pulleys were manufactured using the profiling method with the application of a two-edge cutting tool, a set of end mills, wire electrical discharge machining and an abrasive waterjet. The obtained results showed that the profiling method, generating method and machining with a two-edge cutting tool are impractical due to the manufacturing errors.

The aim of this work was to show importance of elaboration of proper CAD 3D modeling methodology for cogbelt pulleys with noncircular envelope. The elaborated methodology of teeth designing of these cogbelt pulleys was verified during experimental investigations. CAD models of these cogbelt pulleys have been used for elaboration of technology for NC machine tools, coordinate measuring machines and optical scanners for geometrical features measurements. These models have also been used for construction of transmission in Abaqus system and analysis of its kinematic and dynamics features.

References

