An Influence of Cracks Location on Instability of a Multi-Member Cantilever Slender Supporting System

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Abstract

In this publication the nonlinear slender cantilever column composed of two rods in which the cracks are present is investigated. The cracks are simulated by means of rotational springs with linear characteristic. Additionally the external compressive load is located at the free end to the structure. In order to predict the static behaviour of the column the boundary problem is being formulated by means of the principle of total potential energy. The results of numerical simulations are concern on influence of the locations of cracks on loading capacity. On the basis of the analysis of the results the most dangerous (lowest loading capacity) regions are found.

Keywords: instability, cracks, bifurcation, static instability criterion, slender systems, column.

1 Introduction

The investigations of cracks in the slender systems are very important part of scientific study. The slender systems are classified as ones with much greater length than cross section area. The main subtype of those systems are columns which are used in the constructions as supporting elements. Apart from such unwanted phenomena like flutter instability, buckling or non-axially applied load the presence of cracks is the one that an engineers must take care of. The appearance of cracks reduces loading capacity and has an influence on dynamic behavior of the structure.

In the studies cracks are divided into always open or breathing ones depending on relation between the amplitude of vibration and static the deflection. In the process of creation of mathematical models the use rotational springs or reduced cross section area have been proposed by Chondros (Chondros, 2001). Sokół and Kulawik (Sokół and Kulawik, 2014) have proposed implementation of genetic algorithms in the studies on slender systems while Sokół and Uzny (Uzny and Sokół, 2014) used
analytical methods in order to obtain the regions of local and global instability. An influence of single cracked element of multi member structure on dynamic behavior of the supporting column was discussed by Sokół (Sokół, 2014). On the basis of the results of numerical simulations the characteristic curves as well as amplitude-vibration frequency relations were presented. In the literature the different methods of crack detection were also discussed despite the ones mentioned above: vibration response method (Batabyal et al., 2009), measurements of natural frequencies (Nandwana and Matti, 1997). The investigations on influence of crack have been performed for single beams of columns and presented in (Anifantis and Dimarogonas, 1999; Chondros and Dimarogonas, 1981; Lee and Bergman, 1994; Binici, 2005). The dynamic behavior of the slender systems is presented on the plane external load – vibration frequency while static one can be discussed as external load – investigated parameters relationship. Presentations are mostly done on 2D planes but in more complex situations the use of 3D plots allows one to show an influence of many parameters on investigated feature.

In this paper the cracked supporting multimember slender system is investigated. The structure is loaded by external compressive Euler’s load placed at the free end of the column. The main scope of the study is to examine an influence of locations of cracks and their size on bifurcation load magnitude. The results of numerical simulations are presented as contour graphs in order to achieve the best method of presentation. This analysis allows one to designate the areas of low and high influence of the cracks on loading capacity as well as the rates of capacity drop.
2 Formulation and solution of the boundary problem

The discussed slender supporting cantilever column is shown in the Fig. 1. Two cracks are being simulated by means of rotational springs with linear characteristics (stiffness $C_w$, $C_z$ respectively). The rods of the structure due to appearance of cracks are divided into four elements. The connection between new elements is realized by means of proper boundary conditions (continuity of transversal and longitudinal displacements, bending moments and etc.) which are presented in this publication. The sum of lengths of new elements can be presented as $l = l_1 + l_3 = l_2 + l_4$. On the free end (point of connection of elements 3 and 4) the compressive Euler’s force is localized (force applied axially with constant line of action).

![Fig 1. Bent axes of investigated system.](image)

While the investigations on static behaviour are taken into account the boundary problem can be formulated by means of principle of total potential energy (static criterion of stability):

$$\delta V = 0$$

The potential energy of the system shown in the Fig. 1 can be expressed as:

$$V = \frac{1}{2} \sum_{i=1}^{4} (EJ_i) \int_{0}^{l_i} \left[ \frac{d^2W_i(x)}{dx^2} \right]^2 dx + \frac{1}{2} \sum_{i=1}^{4} (EA_i) \int_{0}^{l_i} \left[ \frac{dU_i(x)}{dx} + \frac{1}{2} \left( \frac{dW_i(x)}{dx} \right)^2 \right] dx +$$

$$+ \frac{1}{2} C_z \left( \frac{dW_1(x_1)}{dx_1} \bigg|_{x_1=l_1} - \frac{dW_3(x_3)}{dx_3} \bigg|_{x_3=0} \right)^2 + \frac{1}{2} C_w \left( \frac{dW_2(x_2)}{dx_2} \bigg|_{x_2=l_2} - \frac{dW_4(x_4)}{dx_4} \bigg|_{x_4=0} \right)^2 +$$

$$+ PU_3 \langle l \rangle$$

(2)
Equating the variation of potential energy to zero (1) which is complemented by the geometrical boundary conditions (3a-g)

\[ W_1(0) = W_2(0) = 0 \]  
\[ \frac{\partial W_1(x_1)}{\partial x_1} \bigg|_{x_1=0} = \frac{\partial W_2(x_2)}{\partial x_2} \bigg|_{x_2=0} = 0 \quad \frac{\partial W_3(x_3)}{\partial x_3} \bigg|_{x_3=l_3} = \frac{\partial W_4(x_4)}{\partial x_4} \bigg|_{x_4=l_4} \]  
\[ W_1(l_1) = W_3(0) \quad W_2(l_2) = W_4(0) \quad W_3(l_3) = W_4(l_4) \]

allows one to obtain:

- natural boundary conditions:

\[ (EJ)_3 \frac{\partial^2 W_3(x_3)}{\partial x_3^2} \bigg|_{x_3=l_3} + (EJ)_4 \frac{\partial^2 W_4(x_4)}{\partial x_4^2} \bigg|_{x_4=l_4} = 0 \]  
\[ (EJ)_3 \frac{\partial^3 W_3(x_3)}{\partial x_3^3} \bigg|_{x_3=b} + P \frac{\partial W_3(x_3)}{\partial x_3} \bigg|_{x_3=b} + (EJ)_4 \frac{\partial^3 W_4(x_4)}{\partial x_4^3} \bigg|_{x_4=l_4} = 0 \]  
\[ (EJ)_1 \frac{\partial^3 W_1(x_1)}{\partial x_1^3} \bigg|_{x_1=l_1} + S_1 \frac{\partial W_1(x_1)}{\partial x_1} \bigg|_{x_1=l_1} + (EJ)_3 \frac{\partial^3 W_3(x_3)}{\partial x_3^3} \bigg|_{x_3=0} + S_3 \frac{\partial W_3(x_3)}{\partial x_3} \bigg|_{x_3=0} = 0 \]  
\[ (EJ)_2 \frac{\partial^3 W_2(x_2)}{\partial x_2^3} \bigg|_{x_2=l_2} + S_2 \frac{\partial W_2(x_2)}{\partial x_2} \bigg|_{x_2=l_2} + (EJ)_4 \frac{\partial^3 W_4(x_4)}{\partial x_4^3} \bigg|_{x_4=0} + S_4 \frac{\partial W_4(x_4)}{\partial x_4} \bigg|_{x_4=0} = 0 \]  
\[ - (EJ)_3 \frac{\partial^2 W_3(x_3)}{\partial x_3^2} \bigg|_{x_3=0} + C_3 \left[ \frac{\partial W_3(x_3)}{\partial x_3} \bigg|_{x_3=0} - \frac{\partial W_1(x_1)}{\partial x_1} \bigg|_{x_1=l_1} \right] = 0 \]  
\[ (EJ)_1 \frac{\partial^2 W_1(x_1)}{\partial x_1^2} \bigg|_{x_1=l_1} - C_3 \left[ \frac{\partial W_3(x_3)}{\partial x_3} \bigg|_{x_3=0} - \frac{\partial W_1(x_1)}{\partial x_1} \bigg|_{x_1=l_1} \right] = 0 \]  
\[ - (EJ)_4 \frac{\partial^2 W_4(x_4)}{\partial x_4^2} \bigg|_{x_4=0} + C_4 \left[ \frac{\partial W_4(x_4)}{\partial x_4} \bigg|_{x_4=0} - \frac{\partial W_2(x_2)}{\partial x_2} \bigg|_{x_2=l_2} \right] = 0 \]
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\[ \left( EJ \right) _2 \frac{\partial^2 W_2 (x_2)}{\partial x_2^2} \bigg|_{x_2 = l_2} - C_w \left[ \frac{\partial W_4 (x_4)}{\partial x_4} \bigg|_{x_4 = 0} - \frac{\partial W_2 (x_2)}{\partial x_2} \bigg|_{x_2 = l_2} \right] = 0 \]  \hspace{1cm} (4h)

\[ S_2 = S_4 \quad S_1 + S_2 = P \quad U_1 (0) = U_2 (0) = 0 \]  \hspace{1cm} (4i, l)

\[ U_1 (l_1) = U_3 (0) \quad U_2 (l_2) = U_4 (0) \quad U_3 (l_3) = U_4 (l_4) \]  \hspace{1cm} (4m, o)

- the differential equation of transversal displacements:

\[ \left( EJ \right) _i \frac{d^4 W_i (x_i)}{dx_i^4} + S_i \frac{d^2 W_i (x_i)}{dx_i^2} = 0 \quad i = 1, 2, 3, 4 \]  \hspace{1cm} (5)

- the longitudinal displacements equation:

\[ U_i (x_i) - U_i (0) = - \frac{S_i}{(EJ)_i} x_i - \int_0^{x_i} \left( \frac{dW_i (x_i)}{dx_i} \right) ^2 dx_i \quad i = 1, 2, 3, 4 \]  \hspace{1cm} (6)

The following forms of solution (7) of the differential equations of transversal displacements (5) are used:

\[ W_i (x_i) = A_i \cos (kx_i) + B_i \sin (kx_i) + C_i x_i + D_i \quad i = 1, 2, 3, 4 \]  \hspace{1cm} (7)

Substituting solutions (7) into boundary condition system of equations has been obtained:

\[ \det \left[ mac_{mn} \right] = 0, \ast 20c \quad m, n = 1, 2, 3, . . . , 16. \]  \hspace{1cm} (8)

Matrix coefficients of system (8) equated to zero is the transcendental equation on the basis of which the an influence of the crack size/location on bifurcation load (loading capacity) of an investigated slender system can be obtained.

3 Results of numerical simulations

All the results of numerical simulations are presented in the non-dimensional form according to the following relations:

- bifurcation load magnitude

\[ p = \frac{P l^2}{(EJ)_1 + (EJ)_2} \]  \hspace{1cm} (9)

- spring stiffness by which represents crack size

\[ c_w = \frac{C_w l}{(EJ)_2 + (EJ)_4}; \quad c_z = \frac{C_z l}{(EJ)_1 + (EJ)_3} \]  \hspace{1cm} (10)

- spring (crack) location
The flexural rigidities of elements are equal \((EJ)_1 = (EJ)_2 = (EJ)_3 = (EJ)_4\). The discussed problem is very complex, due to large number of results, in this publication only the small sample of them is presented. At each location described \(0.01 < d_w < 1\) and \(0.01 < d_z < 1\) the following sizes of cracks are plotted: \(c_z = 0.1\) and \(c_z = 0.1, 0.5, 1, 5, 10\). The use of 3D plots allows one to present a relation between all investigated parameters.

**Fig 2.** Influence of locations of cracks on bifurcation load magnitude; other data \(c_z = 0.1, c_w = 0.1\).
**Fig 3.** Influence of locations of cracks on bifurcation load magnitude; other data $c_z = 0.1$, $c_w = 0.5$.

**Fig 4.** Influence of locations of cracks on bifurcation load magnitude; other data $c_z = 0.1$, $c_w = 1$. 
Fig 5. Influence of locations of cracks on bifurcation load magnitude; other data $c_z = 0.1$, $c_w = 5$.

Fig 6. Influence of locations of cracks on bifurcation load magnitude; other data $c_z = 0.1$, $c_w = 10$.

An analysis of the results shown in the Fig. 2 allows one to conclude that if the cracks sizes are equal the distribution of bifurcation load on the $d_z - d_w$ plane is symmetrical. The lowest loading capacity ($p = 0.19$) can be found when both cracks are localized near the fixed end of the structure while the highest ($p = 2.46$) near the free one. It shows the area at which crack appearance has small influence on bifurcation load magnitude. When the cracks are different (Fig. 3) the plot is
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unsymmetrical. The area of low crack influence on bifurcation load is greater than the one shown in Fig. 2. Also the lowest bifurcation load has risen up to \((p = 0.47)\). The change of crack location according to \(d\) parameter shows that the crack in the other element can be “placed” lower what results in higher bifurcation load magnitude than in the Fig. 2. Together with the reduction of the crack size in the internal member (increase of the rotational spring stiffness) the reduction of an influence of the crack location on bifurcation load magnitude can observed. The lowest loading capacity is increasing as well as the size of the area at which the crack presence has small influence on investigated parameter’s magnitude (see Fig. 4, 6). It can be concluded that for each investigated configuration the area of small influence of cracks can be found as well as the locations of cracks at which the difference between minimum and maximum loading is high, furthermore this change may be rapid.

4 Conclusion

In this paper the geometrically nonlinear supporting cantilever slender structure composed of two continues rods in which the cracks appear has been presented. The simulation of cracks is realized by implementation of rotational springs with linear characteristics in the points of cracks presence. The column is loaded by external compressive Euler’s force. An analysis of the data obtained during the numerical simulations processes allows one to concluded that:

- the point of crack appearance has great influence on loading capacity of the structure; when crack appears near the free end the loading capacity reduction is smaller than if it is localized near the fixed one,

- when sizes of both of cracks are equal the bifurcation load distribution is symmetrical,

- in the case when one of cracks is small (high rotational spring stiffness) and the other is big the reduction of bifurcation load magnitude is small due to acquisition of external load by the stiffer element (the element in which the crack is placed higher from the fixed end),

- bifurcation load varies through the length of the column,

- the obtained data during the simulation process can be used in creation of data sets used in structure monitoring.

In the future the investigations on dynamic characteristic (3D plots external load – vibration frequency – crack size, shape modes) must be done due to complexity of the problem.
Acknowledgements

The study has been carried out within the statutory funds of the Czestochowa University of Technology (BS/PB-1-101-3020/11/P).

References


